

Optimizing Yield Curve Positioning for Multi-Asset Portfolios

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Executive Summary

- Many investors buy U.S. Treasuries to hedge equity risk. But do long Treasuries deliver the optimal risk/return trade-off?
- Our research investigates this question by analyzing the optimal allocation of duration along the yield curve.
- We find that a dynamic swap overlay strategy – positioned along the yield curve to account for carry and the stage of the business cycle – has the potential to deliver sizable Sharpe ratio and drawdown improvements.
- On average, the “belly” of the curve – around five years – maximizes the diversification benefit relative to a benchmark portfolio.

Investors often justify the allocations to long Treasuries in their portfolios by noting that these allocations have historically offered positive returns and diversification of equity risk. We expand on this idea by investigating the particular point on the yield curve where an investor obtains the best combination of return and diversification. We set forth a dynamic swap overlay strategy to position along the curve, depending on the carry and roll-down, and the position of the economy across the business cycle. The dynamic swap positioning delivered sizable Sharpe ratio and drawdown improvements. On average, the intermediate level of duration – around five years – maximized the diversification benefit compared

with a benchmark portfolio. This framework may be particularly relevant for managers that do not have an exact liability matching or active leverage constraint.

INTRODUCTION

If you are an investor with exposure to equities, in all probability your portfolio contains some form of diversifier of equity risk. A widely accepted way to offset equity risk is to add bonds because of their propensity for negative correlation and positive yield. Within the fixed income asset class, Treasuries and interest rate swaps can offer hedging power without exposing the portfolio to industry credit risk or significant counterparty risk. If you are

interested in maximizing the bang-for-your-buck of equity hedging benefits along the yield curve, how should you evaluate the risk/return trade-offs of using duration to hedge against equity risk?

In recent decades, government bonds and swaps have exhibited a negative correlation with equities that has served to strengthen the case for allocating risk to these financial instruments. Furthermore, regardless of the mathematical sign of the correlation (positive or negative), Treasury bonds' excess returns have been positive in nine of the past 10 recessions. This feature of bonds – being a positive return diversifier of equity risk – is a mainstay of strategies like risk parity, which, under certain correlation and expected return conditions, can be better than a traditional market-cap-weighted portfolio when leverage constraints are relaxed (Asness et al. 2012).

While the role of government bonds and their associated interest rate swaps as a hedge to equity risk is well accepted in modern portfolios, one question that remains open is the particular point on the yield curve where an investor obtains the best combination of return and diversification benefit. This paper attempts to provide a simple framework for answering this question in the context of both a standard 60/40 portfolio of stocks and bonds and a risk-balanced portfolio consisting of equal risk allocations to bonds and equities. We found that the optimal point on the curve changes based on several factors, including whether the investor is using Treasuries or swaps, the steepness of the yield curve and the dynamic nature of the correlation between stocks and bonds.

RELEVANCE FOR PUBLIC PENSIONS

Based on a panel of publicly available data (Willis Towers Watson 2018), the median public pension plan in the seven most important pension markets (Australia, Canada, Japan, Netherlands, Switzerland, the U.K. and the U.S.) in 2017 had roughly 46% in equities, with the balance of 54% allocated to lower-volatility and/or diversifying assets such as fixed income, alternatives and cash. In a risk context, most of the variation of returns was driven by the allocation to public or private equity and alternatives, while only a small fraction was driven by the remaining portfolio allocations. Based on this risk assessment, the fraction allocated to equities is one of the most important decisions a public pension plan makes, as this drives a large fraction of the overall risk. Ultimately, the large allocation to equity is driven by the need to deliver upon expected returns over the long run (typically 7.5% for a public pension fund).

Another important decision a public pension plan makes is how to allocate the remaining 30% of risk to nonequity assets. This bucket typically consists of some combination of fixed income and alternatives, such as hedge funds, that is constructed to seek to deliver positive returns yet exhibits low correlation to equities. In times of severe stress in equity markets, this under-allocation could offset some losses and provide a hedge to equity risk; in normal times, it could serve as a tailwind to returns to help achieve the return target. In a separate paper, we discuss how to optimize a basket of these strategies (Baz, Davis and Rennison 2017). Several strategies are generally thought to provide a good basis for this allocation: long Treasuries within fixed income, and trend-following and global macro strategies within the alternatives space.

In this paper, we turn our attention to the fixed income allocation of a representative large pension fund. In particular, we focus on the Treasury exposure, which can be approximated by a long Treasury index. Treasuries are among the most liquid instruments; carry minimal credit risk; have been, on average, negatively correlated with equities for the past 20 years; and have historically offered positive yields – all key factors supporting their inclusion in equity-heavy portfolios. Interestingly, public pension plans have chosen to target their Treasury allocations to longer maturities, perhaps because many face leverage constraints. Although this may be the optimal point on the curve at certain times, we will show that historically this has not been the optimal point for maximizing returns per unit of duration. On the contrary, the optimal point has been moving across time, depending on the economy's position along the business cycle. A simple framework incorporating aspects like carry and relative value captures the attractiveness of different points on the yield curve and can be used to more optimally obtain duration exposure.

FIRST GLANCE: EXPLORATORY ANALYSIS OF EQUITY BETAS

One simple way to analyze hedging effectiveness along the curve is to take every point of the curve scaled by duration and calculate its hedging beta with respect to the equity market. The lower the beta, the better hedging properties it will provide.

The appendix shows the results of a simple regression analysis on the beta to the equity market of different points along the curve. First, we focus on zero-coupon Treasury returns. Maturities of between four and seven years provide the best hedge to equities. The “belly” of the curve offers the best beta per unit of duration for the period January 1960 to December 2018 in

a monthly frequency. If we extend the analysis to the swap curve across several developed market countries, we obtain results in the same spirit, with some differences. The absolute value of the beta is maximized at an intermediate maturity for the U.S., Europe and the U.K., although it shifts to the 10-year contract. The Japanese swap market, in contrast, exhibits greater diversification benefit per unit of duration at the 30-year contract, reflecting the unique dynamics of the Japanese bond market.

The analysis centered on beta has obvious limitations, as it does not account correctly for the volatility of each contract. What good is a contract with a negative beta if it is also very volatile? We would be buying a long hedge position at the expense of increased volatility. Because of this, in the next section we present a framework that takes into account the global improvement in the Sharpe ratio derived from shifting the position along the curve.

EXPECTED RETURNS: CARRY AND ROLL-DOWN IN SWAP CONTRACTS

Beyond the discussion of the beta exposure to equities, we should analyze the best dynamic allocation of swap contracts, taking into account not only these diversification benefits but also the estimated expected return and volatility of the strategy. A useful starting point is the self-evident statement that the return r_t of any asset can be decomposed into its expected $E(r_t)$ and unexpected ϵ_t^r components:

$$r_t = E(r_t) + \epsilon_t^r. \quad (1)$$

We use this decomposition to start our analysis by discussing the carry plus roll-down component of the expected return of these instruments. Within the expected return component $E(r_t)$, the carry plus roll-down excess return can be thought of as the excess return of an asset, assuming that market prices stay constant, or $E^C(r_t)$. You could think of carry plus roll-down as the part of the expected return that comes simply from the passage of time. Hence, by definition, carry is a value that can be observed in advance. The second part of the expected return comes from expected price appreciation $E^{\Delta P}(r_t)$:

$$E(r_t) = E^C(r_t) + E^{\Delta P}(r_t). \quad (2)$$

The above decomposition has hidden simplifications that deserve to be made explicit. An important one is that the models used to capture the expected return under constant prices and the expected return under price changes must be correctly identified. If so, the decomposition is an identity. Omissions to the price appreciation model will be irrelevant in our framework if

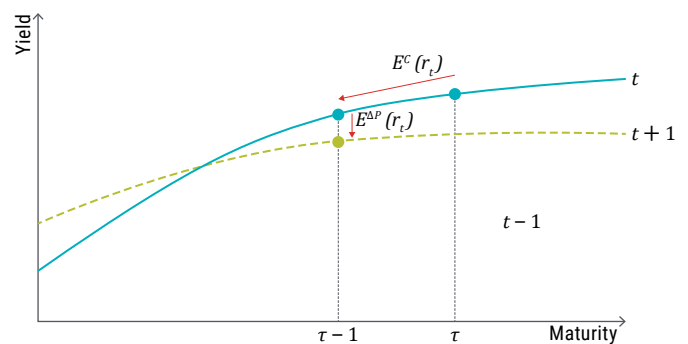
they are evenly applied to all swap tenors. Indeed, given that we are focusing on the comparison of expected Sharpe ratio signals using a particular model, if the same term is omitted in two given tenors the difference will remain constant, leaving the relative ranking untouched.

When applied to bond returns, we define carry as the return assuming that the entire term structure of rates stays constant. Under this definition, carry becomes the sum of the excess yield over the risk-free rate (slope) plus the roll-down derived from stepping from one maturity on the curve to a shorter maturity. If $P_t^T(y_t^T)$ is the price of a bond at time t with maturity T when the yield for maturity T at time t is given by y_t^T and r_t^f is the risk-free rate, then we calculate carry plus roll-down C_t as

$$C_t = y_t^T - r_t^f + \frac{P_{t+1}^{T-1}(y_t^{T-1}) - P_{t+1}^{T-1}(y_t^T)}{P_t^T}. \quad (3)$$

When estimating expected returns, we model as the sum of two parts: the anticipated return for constant market prices (carry plus roll-down $E^C(r_t)$) and the expected return coming from returns converging to their mean values $E^{\Delta P}(r_t)$. Exhibit 1 shows a graphical decomposition between these two parts. This assumes that all the predictable component of $E^{\Delta P}$ comes from mean reversion. To model this mean reversion of bond returns, we consider the dynamic convergence of the yield curve to its expected value as given by an autoregressive model.

Exhibit 1: Return decomposition between carry plus roll-down and repricing



For illustrative purposes only. Source: PIMCO

When expected returns are modeled using the decomposition between carry plus roll-down and curve mean reversion, the curve steepness becomes the focus of analysis. The slope of the yield curve, which gives the implied bond risk premium, is related to the term premium by its own definition. Fama and Bliss (1987) document the predictability of bond returns by the spread between forward rates and one-year rates; Cochrane and Piazzesi (2005) refine this to model bond returns with a measure related to the concavity of the yield curve.

The yield curve tends to flatten late in the business cycle. This movement occurs in tandem with the central bank raising rates beyond expectations in order to curb inflation. The yield curve will steepen when a recession hits, as the short end will reflect an easing monetary policy. This dynamic can serve as a basis for implementing a basic mean-reversion model to capture the portion of expected returns driven by a change in prices, $E^{\Delta P}(r_t)$.

MAXIMIZING CARRY AND ROLL-DOWN IN THE TIME SERIES

Based only on carry and roll-down, the maximum average return is located at the five-year portion of the curve. In line with the current results, carry plus roll-down seems to pinpoint the duration range with the best performance. Exhibit 2 shows the average return collected from carry plus roll-down per unit of duration for each point in the curve analyzed here. Exhibit 3 illustrates the time series of carry and roll-down per year of duration for the time period 1995–2018.

Exhibit 2: Average carry plus roll-down across vanilla interest rate swap contracts

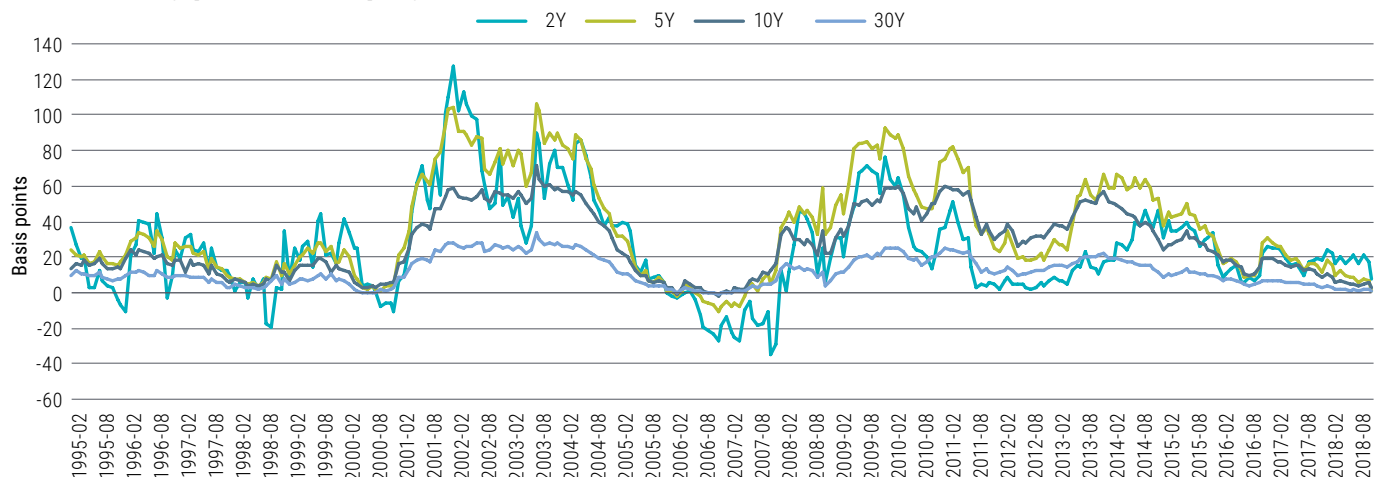
Average carry + roll-down

2-year	5-year	10-year	30-year
34	39	27	11

For illustrative purposes only. Source: PIMCO and Bloomberg as of 31 December 2018. Average carry plus roll-down (in basis points) for different tenors of U.S. interest rate swap contract returns adjusted per year of duration.

The picture painted by the carry plus roll-down time series is more nuanced. Carry has fluctuated along different maturities depending on the shape of the swap curve. In 86% of the sample period 1995–2018, either the two-year or the five-year point maximized carry plus roll-down. The most prolonged period of higher carry on the long end of the curve was between March 2006 and May 2017, corresponding to a late-cycle period with an inverted yield curve.

Exhibit 3: Carry plus roll-down per year of duration



Source: PIMCO and Bloomberg as of 31 December 2018. Carry plus roll-down time series (in basis points) for different tenors of U.S. interest rate swap contract returns adjusted per year of duration.

MEAN REVERSION OF THE YIELD CURVE AND THE ECONOMIC CYCLE

Our mean-reversion model consists of an AR(1) equation projected on a horizon of 12 months in an expanding window. We use a minimum window of 10 years of data to initialize the expanding window. Although modeling the swap curve directly would most accurately characterize the mean reversion at play in this trade, there is a large difference of data availability for

Treasuries versus swaps. The swap curve typically is shaped similar to the Treasury yield curve, and, as Exhibit 4 indicates, the slopes of both curves are highly correlated. Consequently, we choose to estimate the mean-reversion model on the Treasury yield curve.

Exhibit 4: The relationship between Treasury and swap curves

Hypothetical example for illustrative purposes only. Source: PIMCO and the Federal Reserve Board as of 31 December 2018. The exhibit shows the close relationship between the slope of the two- over the 10-year Treasury versus the slope along the swap curve for the same tenors. The swap 2/10 slope has been calculated by subtracting the plain-vanilla interest rate swap rates at the 10 and two tenor, while the Treasury 2/10 slope subtracts the Treasury yields computed by Gürkaynak et al. (2006) for zero-coupon bonds of those respective maturities. The yield basis is annual.

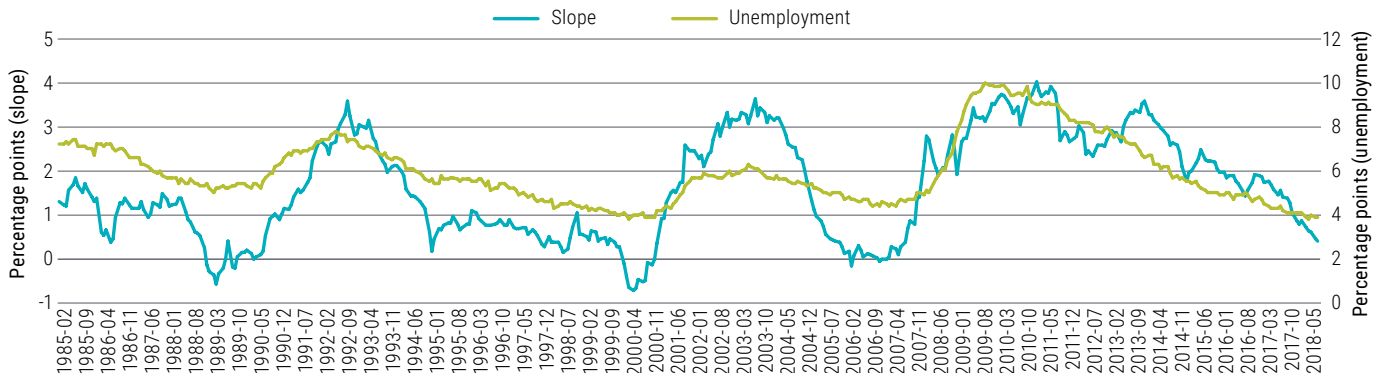
Our AR(1) model is taken on the slope of two-year, five-year and 10-year over the 30-year point on the yield curve and takes the form

$$y_t^{30} - y_t^n = \beta_0 + \beta_1(y_{t-1}^{30} - y_{t-1}^n) + \epsilon_t^n. \quad (4)$$

The slope of the yield curve historically has been a relatively reliable leading indicator of recessions and the economy's position in the business cycle. The long end of the curve reflects long-term expectations about the monetary policy rate. Given that nominal rates comprise expectations of both real rates and inflation, the long end will be informative about the beliefs about long-term inflation and the economy's growth rate. A sharply

positively sloped curve implies an expected upward turn for the economy, as it indicates the bottom of the contraction of the cycle and the beginning of the recovery.

Exhibit 5 shows the time series of the slope of the 30-year over the two-year Treasury bond versus the U.S. unemployment rate. This plot underlines two observations. First, there is a clearly identifiable cycle in the slope of the curve that suggests a mean-reversion pattern in this market. Second, the larger oscillations in the slope cycle are tied to the economic cycle, with rises in unemployment associated with steepening of the curve, and vice versa. The sample correlation of slope and unemployment over the period 1985–2017 amounts to 71%.

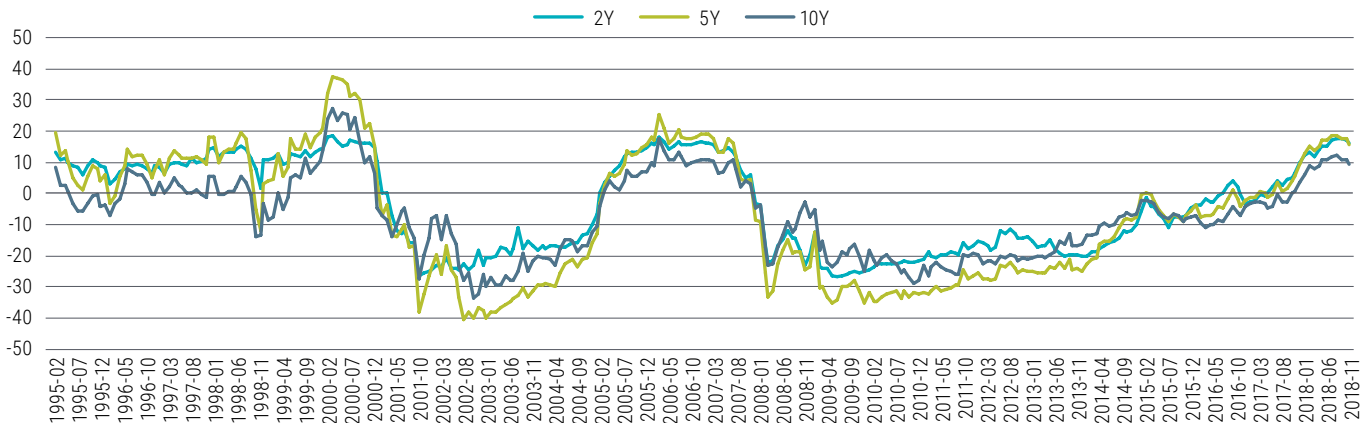
Exhibit 5: Yield curve slope versus unemployment rate

Source: PIMCO and Haver Analytics as of 31 December 2018. The green line shows the headline unemployment rate, which is measured on the right vertical axis (in percentage points). The blue line shows the slope of the yield curve between the two-year and 30-year points, measured on the left vertical axis.

The forecast derived from this model is implemented by rebalancing to one year of duration in the swap at the end of the month. The return time series coming from mean reversion per unit of duration is plotted in Exhibit 6. We add this mean reversion to our strategy by moving into the portion of the curve

where the expected return derived from mean reversion is the highest. We compare these returns by calculating the returns obtained by betting on the expected change in the slope of the two-year, five-year and 10-year points of the yield curve versus the 30-year point.

Exhibit 6: Expected return from mean reversion



Hypothetical example for illustrative purposes only. Source: PIMCO and the Federal Reserve Board as of 31 December 2018. The expected return from the mean-reversion signal is calculated by projecting the fitted AR(1) model described by Equation 4 into a horizon of 12 months. We assume that the 30-year point remains unchanged and infer the return on each swap contract based on the forecasted slope. The expected return is reported in basis points on an annual basis.

The mean-reversion model presented in this paper captures only the business-cycle factor, in a naive manner. Sophisticated models for bond risk premia in general and for the swap curve in particular must explicitly tackle the more important challenges of modeling the swap spread itself and the speed of mean reversion across different business cycles.

OPTIMAL DURATION POSITIONING IN THE CONTEXT OF A REPRESENTATIVE PUBLIC PENSION FUND

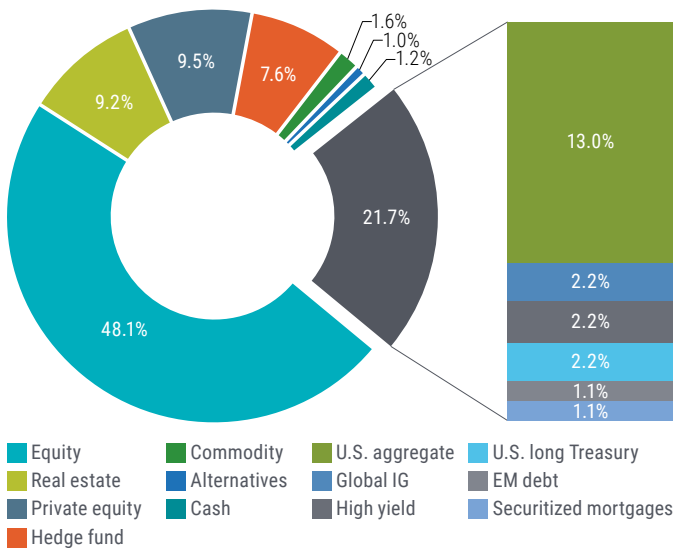
In the context of a large pension fund, we are interested in both the expected return and the volatility of a representative portfolio that includes a swap overlay to gain exposure to duration, as well as the particular hedging properties of the curve location with respect to this benchmark portfolio. Exhibit 7 shows the average asset distribution of U.S. pension plans according to their size. Pension fund allocations do not vary greatly across fund size; the most notable divergence is a greater private equity exposure for the largest funds. The breakdown for the main asset classes is almost identical. In large pension funds, with assets greater than \$50 million, slightly under half of the allocations are directed toward the equity market and 22% are allocated to fixed income instruments.

Exhibit 7: Average allocations of pension plans according to size

Asset allocation	Pension plan size		
	≤ 5M\$	≤ 50M\$	> 50M\$
Equity	49%	50%	48%
Fixed income	24%	22%	22%
Real estate	7%	6%	9%
Private equity	5%	8%	10%
Hedge fund	7%	6%	8%
Commodity	4%	3%	2%
Alternatives	2%	2%	1%
Cash	1%	2%	1%
Other	1%	0%	0%
Total	100%	100%	100%

Source: PIMCO and the Center for Retirement Research at Boston College as of 31 December 2018

Exhibit 8 shows a decomposition of an average pension fund portfolio, and a detail on the fixed income allocation. Fixed income is allocated approximately 60% to U.S. aggregate, 10% to global investment grade, 10% to high yield, 10% to U.S. long Treasuries, 5% to emerging market debt and 5% to securitized mortgages. These allocations are distributed into the total 21.7% fixed income in the total portfolio, resulting in the percentages shown in the exploded pie chart.

Exhibit 8: Average pension plan allocation to asset classes

Source: PIMCO, Bloomberg and the Center for Retirement Research at Boston College as of 31 December 2018. The proxy for equities is the SPX TR Index. We use the Bloomberg Barclays US Aggregate Bond Index, J.P. Morgan Global Aggregate Bond Index, Bloomberg Barclays US Corporate High Yield Total Return Index Value Unhedged USD, Bloomberg Barclays US Treasury Index, Bloomberg Barclays Emerging Markets Hard Currency Aggregate Index, S&P 500 Real Estate Funds Broad Total Return, S&P 500 Private Equity Index, HFRI Fund Weighted Composite Index, Bloomberg Commodity Index and Morningstar Diversified Alternatives Index to approximate the returns of the U.S. aggregate, global investment grade, high yield, long Treasury, emerging markets debt, real estate, private equity, hedge fund, commodity and alternatives, respectively. The proxy for cash is one-month LIBOR, and the proxy for securitized mortgages is the Bloomberg Barclays US MBS Index Total Return Value Unhedged.

To evaluate the optimal contract used to obtain the target duration exposure, we consider the following exercise: We replace the 10% U.S. long Treasury within the fixed income allocation with a Libor exposure and add a swap overlay leveraged to obtain the same duration as a representative U.S. long Treasury index. Given that we aim to determine which position in the swap curve provides the maximum benefit to the portfolio, we will calculate the expected Sharpe ratio to serve as a ranking criterion for different contracts across time. Each month (see appendix for quarterly rebalancing), we select the swap tenor with the highest expected Sharpe ratio to optimally position along the curve.

The analysis that follows relies on leveraging swap contracts at a particular point of the yield curve. For that reason, this framework tends to be most relevant for strategies that are not heavily constrained on leverage positions. Likewise, we choose to compare the strategy with the same level of duration

achieved by the benchmark Treasury portfolio so we can assess the effectiveness of each contract at reaching the same target. A separate and very interesting question is the amount of duration to take to offset equity risk – a question we do not answer in this exercise.

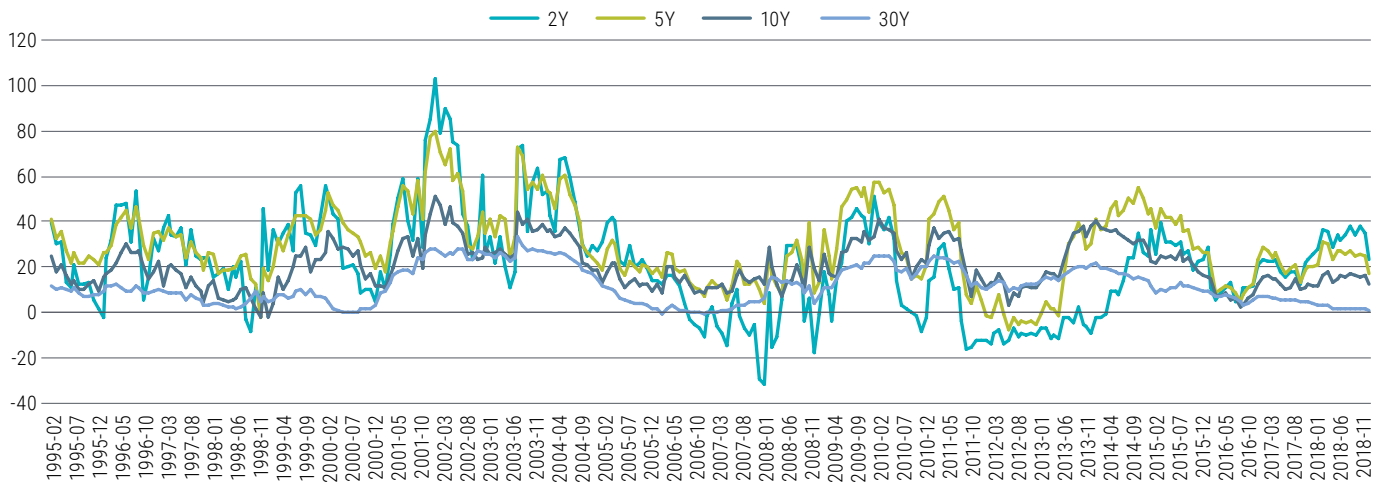
AN OPTIMAL SWAP TENOR FRAMEWORK: EXPECTED SHARPE RATIO MAXIMIZATION

First, we estimate expected returns. We characterize expected returns by adding the carry plus roll-down $E^C(r_t)$ to the expected return coming from price changes $E^{\Delta P}(r_t)$, which in our framework is uniquely informed by the mean-reversion model. When we add both effects, we obtain the resulting time series of expected returns for each maturity, shown in Exhibit 11. Total expected returns exhibit some cyclicity, with synchronized lows in the periods immediately before the dot-com and financial crisis recessions. The volatility of the front end of the curve is, in general, larger than that of the long end. On average, the front and intermediate sections of the curve provide the highest expected return per unit of duration among the considered duration levels, although there is considerable variability across time. If we decompose our sample among the precrisis period 1995–2007, the financial crisis period 2008–2011 and the post-financial-crisis period 2012–2018, we see that the difference in favor of short-tenor-swap average returns widens almost fourfold during the financial crisis and reverses in the period after the crisis, when, on average, the highest return is located in the 10-year contract (see Exhibit 9).

Exhibit 9: Average returns and volatility of different swap tenors

		2-year	5-year	10-year	30-year
1995–2007	Mean	0.18%	0.17%	0.13%	0.10%
	Volatility	0.57%	0.78%	0.62%	0.43%
2008–2011	Mean	0.49%	0.36%	0.24%	0.19%
	Volatility	0.44%	0.46%	0.71%	0.90%
2012–2018	Mean	0.03%	0.11%	0.13%	0.10%
	Volatility	0.29%	0.24%	0.30%	0.43%
Total sample mean		0.58%	0.64%	0.54%	0.43%
Total sample volatility		1.02%	0.86%	0.75%	0.75%

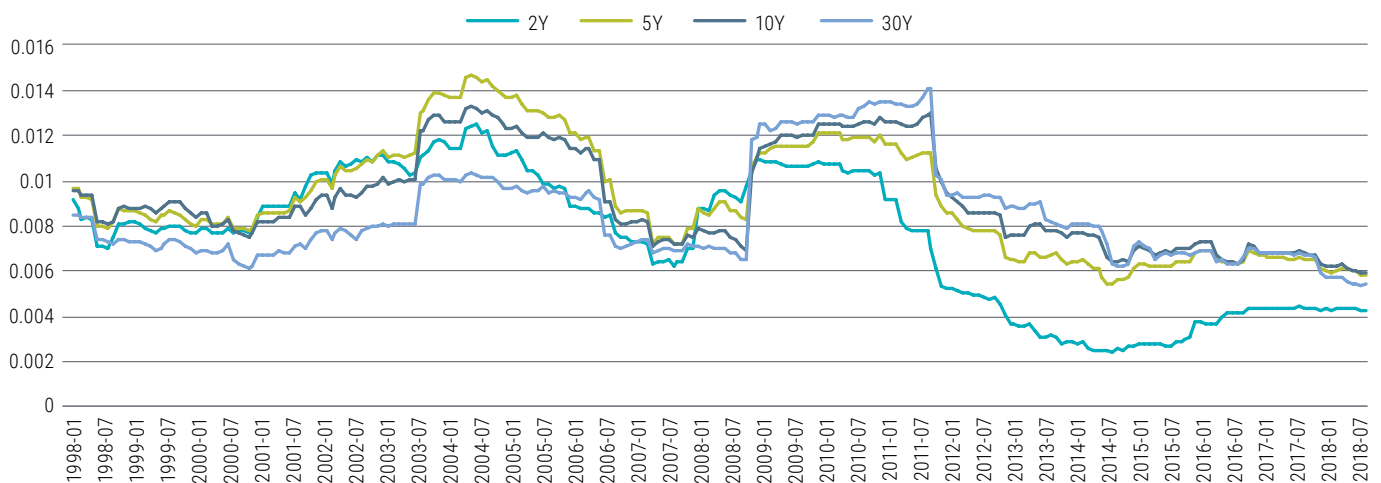
Hypothetical example for illustrative purposes only. Source: PIMCO and Bloomberg as of 31 December 2018. The mean return corresponds to the annualized average monthly returns of the respective swap contract for the time period specified in the first column. Volatility corresponds to the annualized sample standard deviation of monthly returns.

Exhibit 10: Total expected return from carry plus roll-down and mean reversion

Hypothetical example for illustrative purposes only. Source: PIMCO and the Federal Reserve Board as of 31 December 2018

To compute the Sharpe ratio corresponding to each point of the curve, we need to include the return of the benchmark portfolio in the numerator. Although a predictive model based on a factor specification could be used to render a better forecast of the expected return of the benchmark, we fix its expectation at a long-term average calculated using the full sample from 1995 through 2018. In this time frame, the average return of the benchmark is 8.1%, which corresponds to the long-term average of the portfolio that results from replacing the U.S. long Treasury component with Libor. The total expected return of the portfolio with swap overlay corresponds to this long-term average plus the expected return model of each swap point in the curve, $E^C(r_t) + E^{\Delta P}(r_t)$. Exhibit 10 plots the time series of the total expected return for each swap tenor.

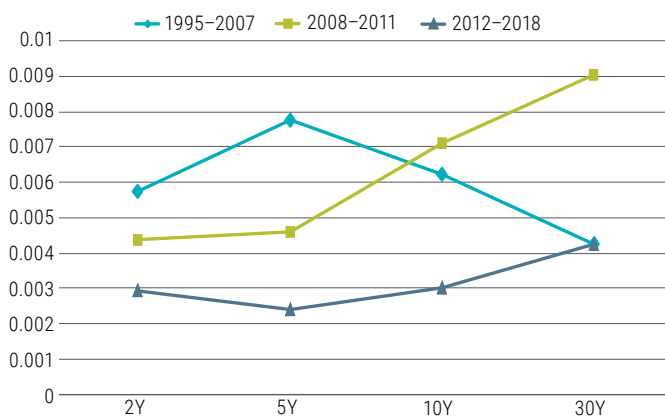
We now turn to the denominator of our projected Sharpe ratio. Given that we are implementing a swap overlay on the benchmark portfolio, the total volatility will be determined by that of the particular swap tenor used to achieve the target volatility, scaled by the multiple needed to achieve the target duration of the benchmark. Differences in volatility among combinations of the benchmark with different points on the curve will arise from 1) differences in the volatility of different swap contracts and 2) different correlations of those contracts with the benchmark. From an econometric point of view, we model portfolio volatility by computing the historical standard deviation of returns of the portfolio with the swap overlay, using an expanding window.

Exhibit 11: Rolling (36-month) volatility swap returns per unit of duration

Source: PIMCO and Bloomberg as of 31 December 2018. Calculations based on the standard deviation of swap returns scaled per unit of duration on a rolling window of 36 months.

(Using a rolling window between 12 and 48 months does not generate significant changes in total performance.) Alternative specifications using more sophisticated volatility modeling, such as a GARCH (1,1) process, yield results with no material improvement in this case. As Exhibit 11 makes apparent, the ranking of volatilities among different swap tenors has changed across time. Although the 30-year contract displays the lowest volatility before 2007, during the post-quantitative-easing (QE) era, after 2009, the two-year contract is the least volatile by a wide margin.

Exhibit 12: Average volatility of duration-adjusted returns across regimes for different swap tenors

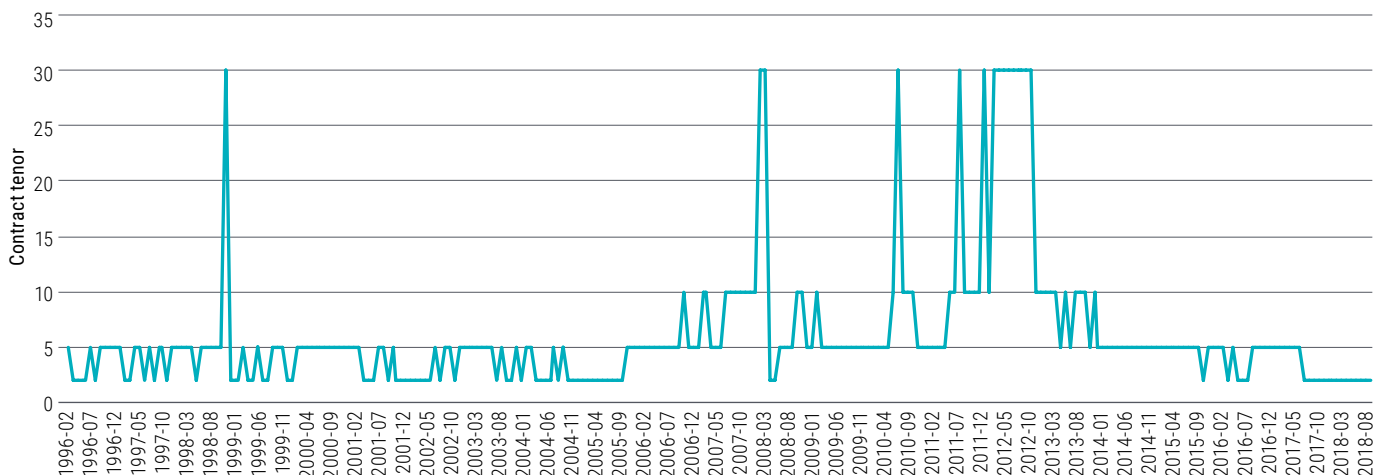


Source: PIMCO as of 31 December 2018. Calculations of the annualized standard deviation of swap returns are adjusted per unit of duration for each specified time interval.

As Exhibit 12 illustrates, volatility profiles across tenors have gone through regime changes over time. From 2008 onward, the 30-year contract has become the most volatile per unit of duration, although the difference with the two-year contract has diminished since the peak in the middle of the financial crisis.

Exhibit 13 shows the optimal swap contract according to the maximum expected Sharpe ratio criterion. In line with the regression evidence presented previously, this criterion indicates that it is optimal to position the portfolio in the five-year tenor 54% of the time, followed by the two-year 28.1% of the time and the 10-year 12.4%. It is remarkable that the front end of the curve was an optimal positioning only during the pre-crisis period and from 2016 onward. In the decade 2006–2015, the two-year contract was chosen only 2.5% of the time. During the period of financial turmoil and subsequent quantitative easing in the decade following 2007, the two-year contract was not optimal according to the yield curve mean-reversion model, which predicts a normalization of the slope to historical levels. One challenge to be tackled by a more sophisticated mean-reversion model is to capture the extraordinarily long mean reversion of the yield curve experienced in the QE era; it greatly exceeds that of previous business cycles. While we can qualitatively describe the exceptional regime of the past decade as related to the quantitative easing policies implemented by the Federal Reserve and central banks around the world, a systematic way to capture this phenomenon would render an obvious advantage to the forecasting power of the expected Sharpe ratio used to switch among contracts.

Exhibit 13: Optimal contract according to dynamic swap strategy

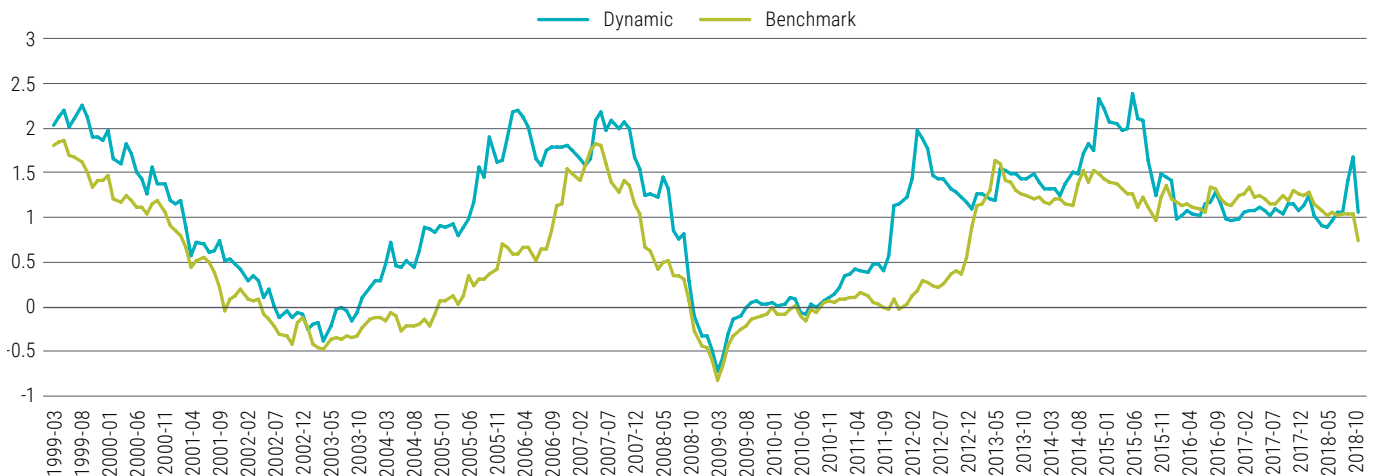


Hypothetical example for illustrative purposes only. Source: PIMCO as of 31 December 2018. This exhibit shows the optimal contract as suggested by the maximum expected Sharpe ratio criterion proposed in this paper. The vertical axis indicates the tenor of the swap contract to leverage to obtain the target duration.

Under what conditions does the optimality of the medium-tenor part of the curve break down? If we experience a structural change in the slope of the curve, either because of a repricing of inflation risk or because of an anticipated new inflation regime, the model will mean revert to the incorrect target. To derive the best benefit from this approach, the manager will have to keep her attention on the correct model of yield curve mean reversion. Different views of the target values of the yield curve will inform different optimal positioning strategies.

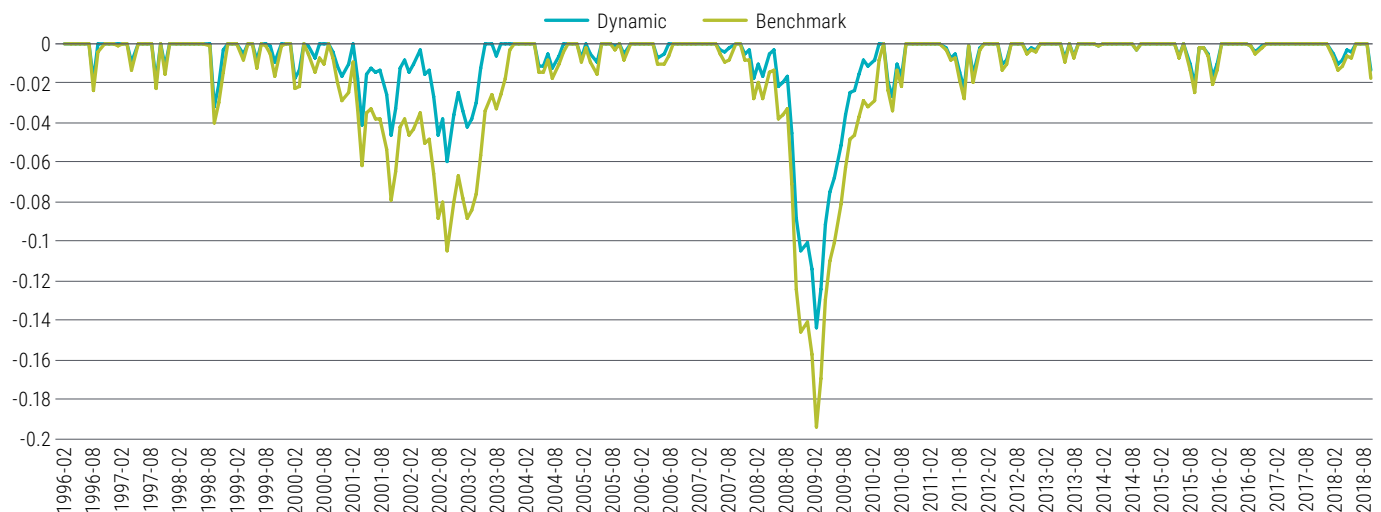
The dynamic swap switching strategy compares favorably with the benchmark, displaying a total Sharpe ratio of 0.915 versus 0.621 in the benchmark pension fund portfolio. Moreover, the 36-month rolling marginal Sharpe ratio over the benchmark, illustrated in Exhibit 14, was positive for 99% of the sample before 2012 and dropped to 63% from 2012 onward. A notable benefit of this strategy is shown in Exhibit 15, where we observe the improved protective effects with respect to those offered by the Treasury bond exposure in the benchmark. As illustrated in the exhibit, the maximum drawdown experienced in both the dot-com crash and the financial crisis is substantially lower in the portfolio with swap overlay.

Exhibit 14: Rolling 36-month Sharpe ratio for portfolio with dynamic swap overlay versus benchmark

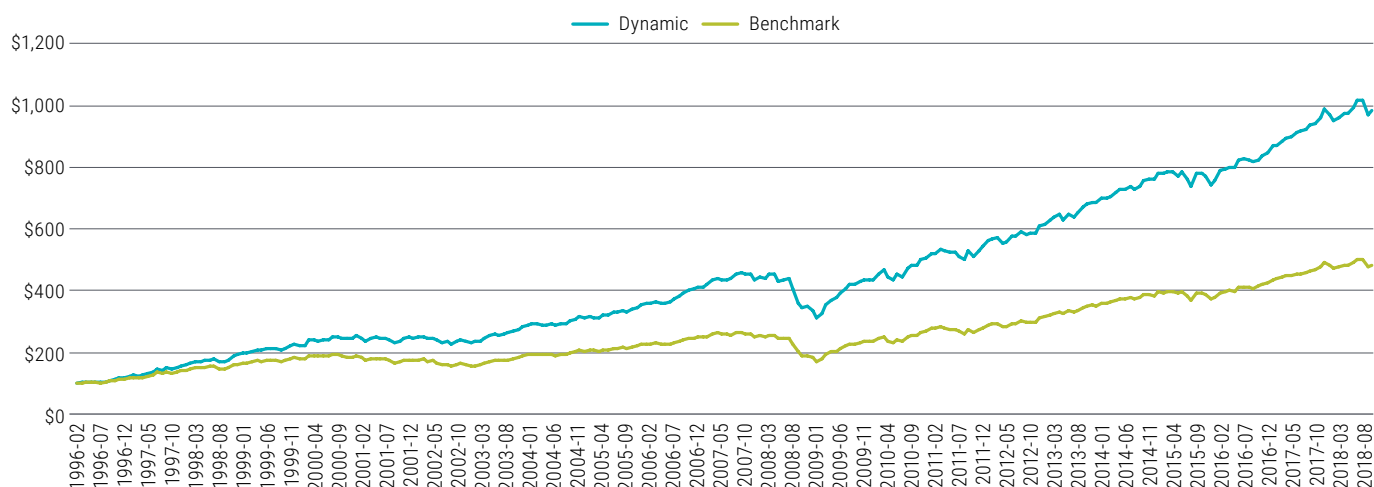


Hypothetical example for illustrative purposes only. Source: PIMCO as of 31 December 2018. We calculate the Sharpe ratio by averaging the annualized excess returns of each portfolio on a rolling window of 36 months. Excess returns are calculated with respect to one-month Libor. The denominator is calculated using the sample standard deviation of the annualized portfolio returns.

Exhibit 15: Drawdown for portfolio with dynamic swap overlay versus benchmark



Hypothetical example for illustrative purposes only. Source: PIMCO and Bloomberg as of 31 December 2018. This exhibit shows the drawdown patterns for the benchmark strategy versus the optimal swap switching ("dynamic") strategy suggested in this paper for the time period 1996–2018. The drawdown is the peak-to-trough decline in the portfolio value, quoted as a fraction, between the peak and subsequent trough.

Exhibit 16: Value of \$100 invested in March 1996 (dynamic swap strategy versus benchmark)

Hypothetical example for illustrative purposes only. Source: PIMCO and Bloomberg as of 31 December 2018. This exhibit shows the drawdown patterns for the benchmark strategy versus the optimal swap switching ("dynamic") strategy suggested in this paper for the time period 1996–2018. The drawdown is the peak-to-trough decline in the portfolio value, quoted as a fraction, between the peak and subsequent trough. Model performance figures do not reflect the deduction of investment advisory fees. Performance would be lower if fees were applied.

CONCLUSION

We have presented a framework for considering the optimal positioning along the yield curve to hedge a generic portfolio against equity drawdowns. By replacing the typical long Treasury exposure in a representative pension fund portfolio with a leveraged swap overlay, we are able to offer an improved Sharpe ratio and drawdown profile. The optimal positioning depends on the carry and roll-down of each contract along the curve, plus a suitable mean-reversion model for different segments of the yield curve.

In this paper, we have presented a simplified mean-reversion model of the yield curve that relies only on autoregressive properties to find the speed of mean reversion. In practice and in our historical sample of interest, we observe substantial variation in the mean reversion of the swap curve, as well as irregularity in the business-cycle duration, the bond market response to it and the dynamics of the swap spread. The astute manager will find it useful to insert more sophisticated models for the mean reversion in the swap curve into the general framework presented here in order to improve the performance and hedging effectiveness of his fixed income exposures.

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APPENDIX

A simplified analysis using GSW zero-coupon data

We first carry out a simplified analysis using data from Gürkaynak et al., henceforth referred to as GSW (2006), to calculate yields for zero-coupon Treasury bonds. We introduce this analysis as a first approximation to the problem because GSW yields are imperfectly linked to tradable assets.

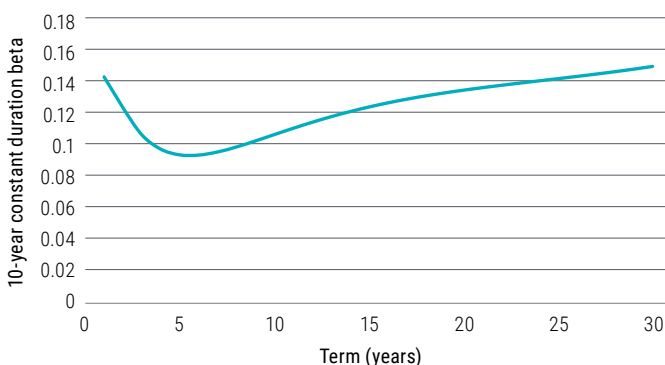
To determine which point of the yield curve provides the best hedge to equities, we normalize the excess zero-coupon bond returns by their analytical duration and estimate the beta regression as

$$\left(\frac{r_i}{D_i}\right)_t = \alpha_i + \beta_i r_{m,t} + \epsilon_t.$$

In this equation, the beta term β_i measures the sensitivity to equity returns of a normalized 10-year-duration portfolio that uses bonds of maturity D_i and cash. For example, a portfolio of 100% allocated to the 10-year point in the zero-yield curve will have a theoretical duration of 10 and exposure to the market of β_{10} . α_i is the intercept of the regression, and ϵ_t is the regression error. We estimate this regression by feasible generalized least squares (FGLS) to correct for heteroscedasticity and autocorrelation in the error series.

The results show that maturities ranging from four to seven years provided the best hedge to equities, while the short and long ends were less effective. As seen in Exhibit 17, hedging is maximized at the lowest beta value, which happens at the “belly” of the curve.

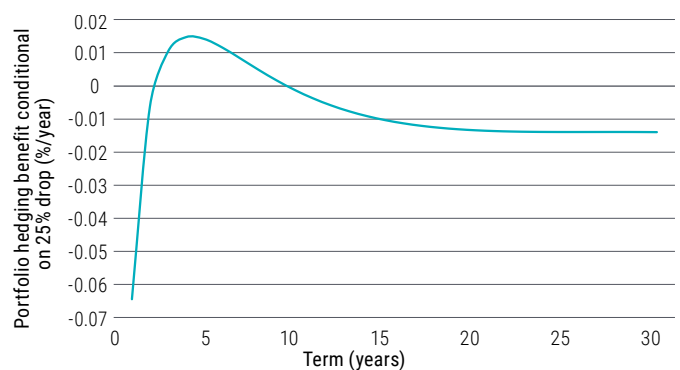
Exhibit 17: Beta of zero-coupon bonds as a function of maturity



Source: PIMCO and the Federal Reserve Board as of 31 December 2018. The exhibit depicts the term of a zero-coupon bond versus the beta obtained in a regression with the returns per unit of duration against the S&P 500 TR index for the period from January 1960 to December 2018, in monthly frequency. The beta has been scaled to match 10 years of duration instead of one year.

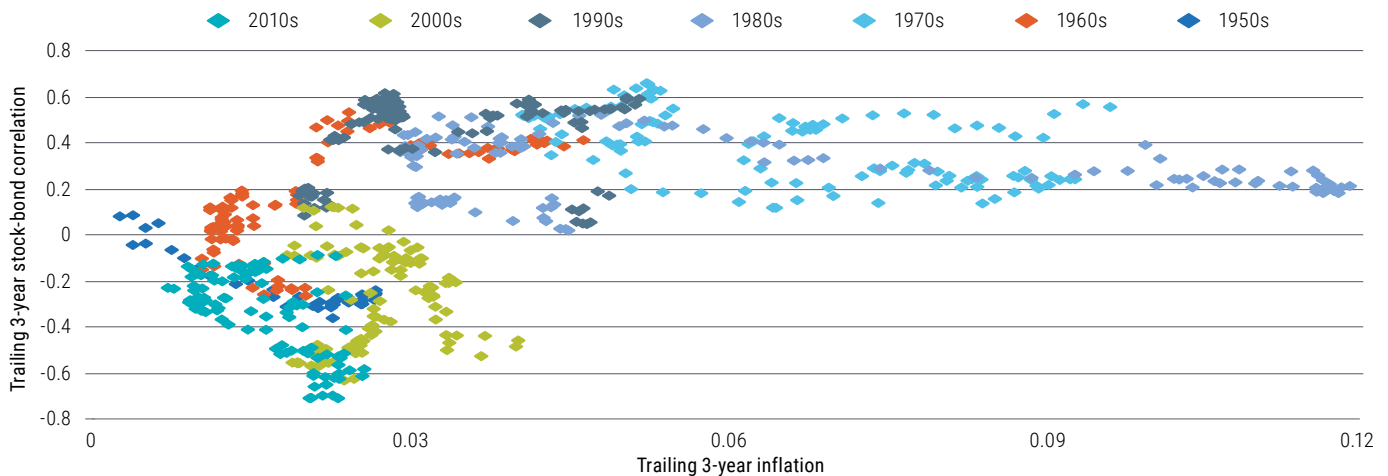
For concreteness, we translate the previous results into a 60/40 portfolio conditional on an equity market drop of 25%. We plot in the horizontal axis the theoretical duration of a zero-coupon Treasury and in the vertical axis the portfolio hedging benefit conditional on a 25% drop in equity prices. The benefit is defined as excess return with respect to the 10-year duration, which is used as the benchmark and therefore corresponds to a benefit of zero. Exhibit 18 shows that being in the duration point between four and seven years will increase our hypothetical portfolio performance by 50 basis points compared with the benchmark.

Exhibit 18: Incremental hedging benefit with respect to a benchmark of a position in a 10-year zero-coupon bond



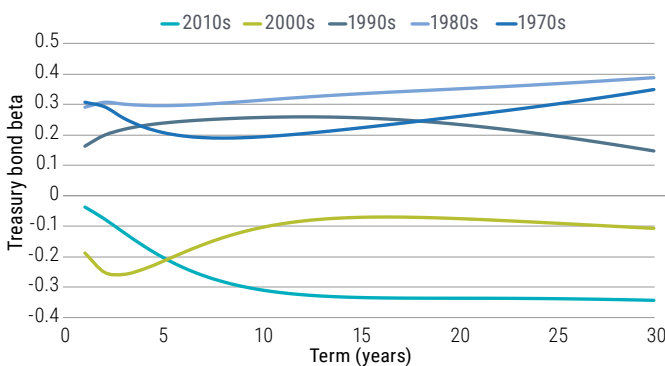
Hypothetical example for illustrative purposes only. Source: PIMCO and the Federal Reserve Board as of 31 December 2018. The exhibit shows the incremental hedging benefit in terms of additional returns obtained by changing the zero-coupon bond used to hedge against equity risk. We perform a simulation of a 25% decline in the stock price index and compare this with the baseline scenario of taking a position in a 10-year zero-coupon bond.

The diversification benefit of Treasury bonds depends crucially on the correlation between stocks and bonds. This correlation has shifted significantly over the decades. Exhibit 19 shows the rolling three-year inflation versus the rolling three-year stock-bond correlation; each decade has been underlined with a different color. What we discover in this exhibit is that the correlation has shifted significantly across the decades, with the 1960s–1990s in the positive range and the new century turning negative. Interestingly, this shift has followed a generalized decrease in inflation levels. Higher average inflation levels were associated with a positive correlation (at decreasing rates); low inflation levels reverted the sign.

Exhibit 19: Trailing 3-year inflation versus trailing 3-year stock-bond correlation

Source: PIMCO and Haver Analytics as of 31 December 2018. The exhibit plots the three-year trailing inflation built on the headline CPI index versus the trailing correlation between the S&P 500 TR index and the return of the 10-year zero-coupon bond, calculated using the methodology of Gürkaynak et al. (2006).

The changing correlation between stocks and equities translates into a different trade-off across bond durations. Thus, while the intermediate maturity ("belly") of the curve minimizes the beta for the 1970s, 1980s and 2000s, this relationship was broken in the past decade. Exhibit 20 plots the beta to the equity market of Treasury bonds along the yield curve for each decade since 1970. Although each previous decade favored either intermediate or short-duration bonds, the analysis for the decade after the last financial crisis shows that the beta is most negative at the longest maturities.

Exhibit 20: Beta of different maturities of Treasury bonds, per decade

Source: PIMCO and the Federal Reserve Board as of 31 December 2018. We compute the returns of zero-coupon bonds on a daily basis to generate the beta profile across different maturities. Betas are calculated with a linear regression containing observations for each respective decade. The data series for the 2010s ends in September 2018.

In conclusion, the four- to 10-year range of the U.S. yield curve has shown the greatest equity diversification benefit. However, while equity beta has been minimized in this range, the sign shifts based on different economic regimes. In particular, the beta of Treasuries to the S&P 500 TR index shifted from positive to negative in the late 1990s, which might reflect increased central bank credibility with respect to inflation.¹ Since 2008, the front end of the curve has been less of an equity diversifier due to post-financial-crisis monetary loosening, which has kept the short rate near the zero lower bound.

Although using zero-coupon bond analysis provides a shortcut to proxy duration effects, using these model-generated yields is not exempt from problems. First, the GSW methodology is difficult to replicate. It is based on filtering out bonds with callable features (Treasury-issued callable bond series before 1985), bonds shorter than three months in duration (they behave "oddly"), Treasury bills (out of concern about segmented markets), 20-year bonds after 1996 (liquidity issues or high coupons decrease demand for tax-related reasons in the U.S.), "on the run" and "first off the run" bonds (liquidity and repo market special status) and ad hoc modifications on a case-by-case basis (example: a May 2013 10-year note was priced at a sustained premium near repo and therefore deleted from the sample). All these ad hoc, case-by-case data selections might make an extension of the GSW methodology to different data frames challenging.

¹ The Federal Reserve formulates policy weighting inflation and output gap. As a result of the successful "conquest of American inflation" (Sargent 1999) and the adoption of inflation targeting, the emphasis on inflation might have decreased. In a low inflation environment, the Fed has shifted its focus to economic activity, providing the market with an equity market put. During the same period, we observed that the correlation between equities and bonds has shifted from positive to negative. Post-2008, the expansive monetary policy implemented with QE and the existence of a zero lower bound on nominal rates have anchored the front end of the curve. In the past decade, the low volatility of short rates decreased their hedging power against equities, as the correlation of any variable with a constant is zero. At the same time, in the past 10 years the low term premium has made the return series of bonds with maturities over 15 years similar to one another.

ANALYSIS WITH SWAP CONTRACTS

The analysis in the previous section serves as only a first approximation to our problem. If we are to use fixed income instruments to hedge against equities, we find swap contracts offer greater market depth for practical implementation: In April 2016, the average turnover associated with interest rate swaps and options was \$2 trillion, versus a quarter of that magnitude associated with government bonds and options. Internationally, more than 27 currencies offer access to active over-the-counter markets for interest rate swaps, whereas only eight have active government bond futures.² Since the 1990s, the evidence has pointed toward a generalized shift of the market benchmark from government bonds to interest rate swaps, possibly motivated by basis risk (Kreicher et al. 2017).

Historically, the swap curve shape has been shaped much like the Treasury yield curve, and the distance between the two (the swap spread) has been positive. Swap spreads represent the markup for the credit spread between interbank lending and the U.S. Treasury. In this section, we move away from nontradable zero-coupon rates to actual quoted on-the-run swap contracts.

This analysis has the advantage of analyzing a more realistic context for a trading strategy, at the cost of shrinking our sample to 1995–2018.

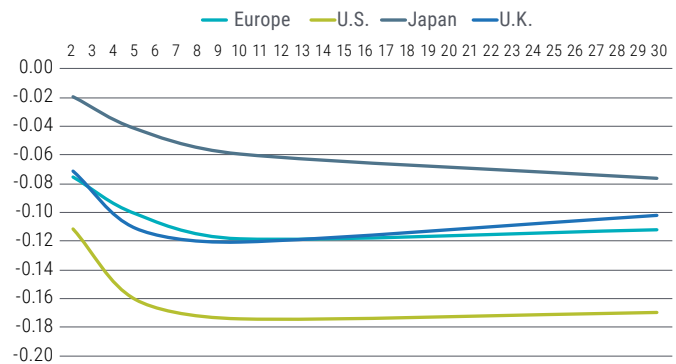
Exhibit 21: Betas of 10-year duration swap returns to the local equity market, 1995–2018

	Pension plan size			
	Europe	U.S.	Japan	U.K.
2-year	-0.075 (0.005)	-0.111 (0.012)	-0.019 (0.003)	-0.071 (0.009)
5-year	-0.101 (0.005)	-0.161 (0.013)	-0.042 (0.003)	-0.111 (0.009)
10-year	-0.118 (0.006)	-0.174 (0.012)	-0.059 (0.004)	-0.121 (0.009)
30-year	-0.112 (0.008)	-0.170 (0.013)	-0.076 (0.005)	-0.102 (0.008)

Source: PIMCO and Bloomberg as of 31 December 2018. The table shows regression betas to swap returns per unit of duration, scaled to a 10-year duration. For each swap tenor, we report the coefficient value and its standard deviation. Values marked in bold indicate the minimum beta in a particular country across different tenors.

Exhibit 21 shows the results of time-series regressions of duration-adjusted swap rates for a universe of four developed market economies (Europe, the U.S., Japan and the U.K.) along the two-, five-, 10- and 30-year contracts on their respective local equity market return. All the estimated market betas are significant at 0.1%. The table also shows the estimated beta coefficient and, below, its associated estimated standard deviation. As Exhibit 22 shows, the absolute magnitude of the hedging beta is maximized at an intermediate maturity for Europe, the U.S. and the U.K. Interestingly, the Japanese swap market exhibits greater diversification benefit per unit of duration for the longest contract, 30 years.

Exhibit 22: Beta per unit of duration in swap contracts, major economies



Source: PIMCO and Bloomberg as of 31 December 2018. The exhibit presents the beta per unit of duration of swap contracts at the two-, five-, 10- and 30-year tenors for four advanced economies.

If we decompose our sample into two subsamples, the first being 1995–2009 and the second 2010–2018, a more nuanced picture emerges. The highest absolute beta per unit of duration in the first part of the sample is always of a shorter maturity than in the second part. In the case of the U.S. and the U.K., the highest-magnitude beta is achieved at the five-year contract, versus the long 30-year contract for the post-financial-crisis era. We can see these results in Exhibit 23, where we show the profile of hedging betas for the four economies in both estimations. As a result of the new low rate environment of the second decade of the 21st century, we observe that the hedging profiles have shifted toward longer durations.

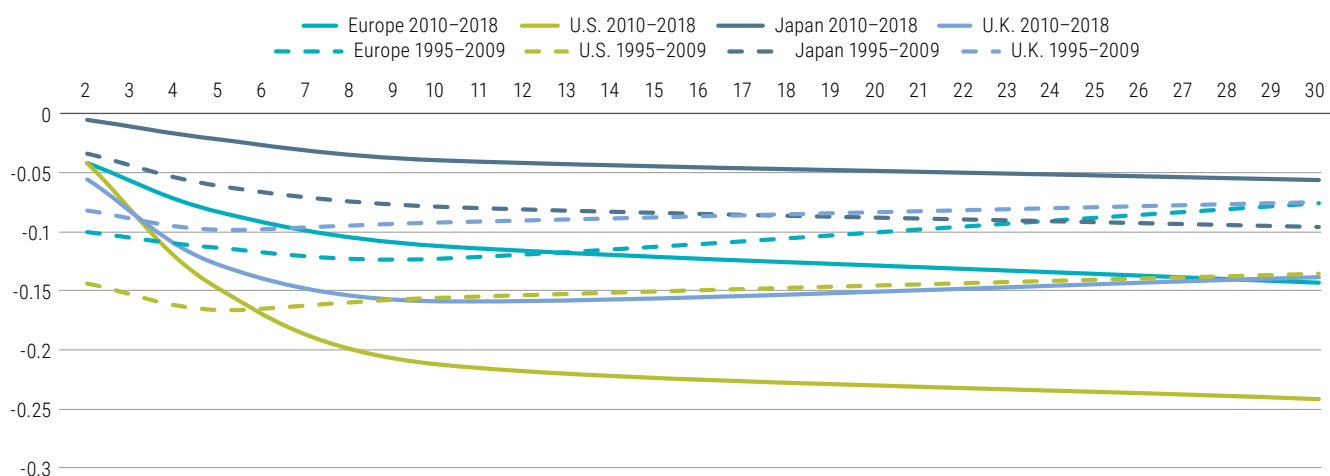
² We define as "active" a market with an average turnover of more than \$1 billion per day.

Exhibit 23: Betas of 10-year duration swap returns to the local equity market

	1995–2009				2010–2018			
	Europe	U.S.	Japan	U.K.	Europe	U.S.	Japan	U.K.
2-year	-0.100 (0.007)	-0.144 (0.017)	-0.034 (0.004)	-0.082 (0.014)	-0.042 (0.006)	-0.042 (0.008)	-0.005 (0.002)	-0.055 (0.008)
5-year	-0.114 (0.007)	-0.166 (0.018)	-0.061 (0.006)	-0.098 (0.012)	-0.084 (0.007)	-0.149 (0.013)	-0.022 (0.003)	-0.128 (0.012)
10-year	-0.123 (0.008)	-0.156 (0.016)	-0.079 (0.007)	-0.092 (0.011)	-0.112 (0.008)	-0.212 (0.015)	-0.040 (0.003)	-0.159 (0.012)
30-year	-0.076 (0.014)	-0.136 (0.017)	-0.096 (0.009)	-0.075 (0.010)	-0.143 (0.009)	-0.242 (0.016)	-0.056 (0.005)	-0.139 (0.011)

Source: PIMCO as of 31 December 2018. The table shows the betas and associated standard deviations of the regression of duration-adjusted daily returns across different advanced economies for two subsamples: 1995–2009 (left) and 2010–2018 (right).

Exhibit 24: Beta of different maturities of Treasury bonds, per decade



Source: PIMCO and Bloomberg as of 31 December 2018. The exhibit presents the beta per unit of duration of swap contracts at the two-, five-, 10- and 30-year tenors for four advanced economies for two subsamples: 1995–2009 and 2010–2018.

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